T3.3 # 14 Resolver por coefs. indeterminados
p. 158

$$y'' - 4y = (x^2 - 3)\,\text{sen}\,2x$$

Sol.

① $y'' - 4y = 0$ $\quad\rightarrow\quad y_c = c_1 e^{2x} + c_2 e^{-2x}$

$(D^2 - 4)y = 0$

$m^2 - 4 = 0 \Rightarrow m_1 = 2$
$\qquad\qquad\qquad m_2 = -2$

caso: raíces reales y distintas

② $y_p = (Ax^2 + Bx + C)\cos 2x + (Dx^2 + Ex + F)\,\text{sen}\,2x$

$y_p' = (Ax^2 + Bx + C)(-2\,\text{sen}\,2x) + (2Ax + B)\cos 2x$
$\qquad + (Dx^2 + Ex + F)(2\cos 2x) + (2Dx + E)\,\text{sen}\,2x$

$y_p'' = (Ax^2 + Bx + C)(-4\cos 2x) + (2Ax + B)(-2\,\text{sen}\,2x)$
$\qquad + (2Ax + B)(-2\,\text{sen}\,2x) + (2A)\cos 2x$
$\qquad + (Dx^2 + Ex + F)(-4\,\text{sen}\,2x) + (2Dx + E)(2\cos 2x)$
$\qquad + (2Dx + E)(2\cos 2x) + (2D)\,\text{sen}\,2x$

componentes:	sen 2x	cos 2x	x sen 2x	x cos 2x	x^2 sen 2x	x^2 cos 2x
$-4y$:	$-4F$	$-4C$	$-4E$	$-4B$	$-4D$	$-4A$
y'' :	$-2B$	$-4C$	$-4A$	$-4B$		$-4A$
	$-2B$	$2A$	$-4A$	$4D$	$-4D$	
	$-4F$	$2E$	$-4E$	$4D$		
	$2D$	$2E$				
$y'' - 4y$:	$-4B+2D-8F$	$2A-8C+4E$	$-8A-8E$	$-8B+8D$	$-8D$	$-8A$
$= (x^2-3)\,\text{sen}\,2x$:	-3	0	0	0	1	0
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
	$-4B+2D-8F=-3$	$-8C=-4E$	$-A=E$	$-B=-D$	$\boxed{D=-\frac{1}{8}}$	$\boxed{A=0}$
		$\boxed{C=0}$	$\boxed{E=0}$	$\boxed{B=\frac{-1}{8}}$		

$\longleftarrow$

$-4\left(\frac{-1}{8}\right) + 2\left(-\frac{1}{8}\right) - 8F = -3$

$+\frac{1}{2} - \frac{1}{4} - 8F = -3$

$\therefore\; F = \left(-3 - \frac{1}{2} + \frac{1}{4}\right)/(-8) = \frac{13}{32}.$

MS, 2010.07.13 (2)

T3.3 #14 cont.

con $A = 0$

$B = -\frac{1}{8}$

$C = 0$

$D = -\frac{1}{8}$

$E = 0$

$F = \frac{13}{32}$

$$y_p = -\frac{1}{8}\cos 2x + \left(\frac{13}{32} - \frac{x^2}{8}\right)\operatorname{sen} 2x$$

$$\therefore \boxed{y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{8}\cos 2x + \left(\frac{13}{32} - \frac{x^2}{8}\right)\operatorname{sen} 2x}$$

T3.3 #36 p. 158 Resolver el PVI

$$y''' + 8y = 2x - 5 + 8e^{-2x}, \quad y(0) = -5, \quad y'(0) = 3, \quad y''(0) = -4$$

① Resolvemos primero $y''' + 8y = 0$

$$(D^3 + 8)y = 0$$

$$m^3 + 8 = 0 \rightarrow m^3 = -8 \rightarrow m_1 = -2$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & 8 \\ & & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

$$m^2 - 2m + 4 = 0 \rightarrow m_{2,3} = \frac{2 \pm \sqrt{4 - 4(4)}}{2}$$

$$m_{2,3} = 1 \pm \frac{\sqrt{12}\,i}{2} = 1 \pm \frac{\sqrt{4 \cdot 3}}{2}\,i = 1 \pm \sqrt{3}\,i$$

$$\therefore \; y_c = C_1 e^{-2x} + e^x (C_2 \cos \sqrt{3}x + C_3 \operatorname{sen} \sqrt{3}x)$$

② Proponemos: $y_p = y_{p_1} + y_{p_2}$

$$y_{p_1} = Ax + B$$

$y_{p_2(0)} = Ce^{-2x}$ pero como ya existe e^{-2x} en y_c

hacemos $y_{p_2} = \boxed{Cxe^{-2x}}$ (equiv. a raíz repetida)

Resolvemos cada caso:

$$y'_{p_1} = A \qquad\qquad 0 + 8(Ax + B) = 2x - 5 = g_1(x)$$

$$y'''_{p_1} = y''_{p_1} = 0 \qquad \therefore \; 8A = 2 \rightarrow \boxed{A = 1/4}, \quad 8B = -5 \rightarrow \boxed{B = -5/8}$$

$$y'_{p_2} = Cx(-2e^{-2x}) + Ce^{-2x}$$

$$y''_{p_2} = Cx(4e^{-2x}) + C(-2e^{-2x}) - 2Ce^{-2x} = 4Cxe^{-2x} - 4Ce^{-2x}$$

$$y'''_{p_2} = 4Cx(-2e^{-2x}) + (4C)e^{-2x} + 8Ce^{-2x} = -8Cxe^{-2x} + 12Ce^{-2x}$$

$\longrightarrow$

T3.3 #36 cont.

	e^{-2x}	xe^{-2x}
Componentes:	e^{-2x}	xe^{-2x}
$8y_{p_2}$:		$8C$
y'''_{p_2}:	$12C$	$-8C$
$y'''_{p_2} + 8y$:	$12C$	0
$8e^{-2x}$:	8	0

$\downarrow$

$$12C = 8$$

$$\therefore \quad C = \frac{8}{12} = \frac{2}{3}$$

$$\therefore \quad y_{p_2} = \frac{2}{3}xe^{-2x}$$

Entonces $\quad y_p = y_{p_1} + y_{p_2}$

$$y_p = \frac{1}{4}x - \frac{5}{8} + \frac{2}{3}xe^{-2x}$$

De lo cual:

$$y = y_c + y_p$$

$$\boxed{y = C_1 e^{-2x} + e^{x}(C_2 \cos\sqrt{3}x + C_3 \operatorname{sen}\sqrt{3}x) + \frac{1}{4}x - \frac{5}{8} + \frac{2}{3}xe^{-2x}}$$

Para obtener C_1, C_2 y C_3:

$$y(0) = -5 \rightarrow \quad -5 = C_1 + C_2 - \frac{5}{8} \qquad (1)$$

$$y'(x) = -2C_1e^{-2x} + e^{x}(-\sqrt{3}C_2 \operatorname{sen}\sqrt{3}x + \sqrt{3}C_3\cos\sqrt{3}x) + e^{x}(C_2\cos\sqrt{3}x + C_3\operatorname{sen}\sqrt{3}x) + \frac{1}{4} + \frac{2}{3}x(-2e^{-2x}) + \frac{2}{3}e^{-2x}$$

$$y'(0) = 3 \rightarrow \quad 3 = -2C_1 + \sqrt{3}C_3 + C_2 + \frac{1}{4} + \frac{2}{3} \qquad (2)$$

$$y''(x) = \frac{d}{dx}\Big[4 + \left(-2C_1 + \tfrac{2}{3}\right)e^{-2x} - \tfrac{4}{3}x e^{-2x}$$
$$+ e^{x}\left(\left[C_3 - \sqrt{3}C_2\right] \operatorname{sen}\sqrt{3}x + \left[C_2 + \sqrt{3}C_3\right]\cos\sqrt{3}x\right)\Big]$$

$$= \left(4C_1 - \tfrac{4}{3}\right)e^{-2x} - \left[\tfrac{4}{3}x(-2)e^{-2x} + \left(\tfrac{4}{3}\right)e^{-2x}\right]$$
$$+ e^{x}\left(\sqrt{3}\left[C_3 - \sqrt{3}C_2\right]\underline{\cos\sqrt{3}x} - \sqrt{3}\left[C_2 + \sqrt{3}C_3\right]\operatorname{sen}\sqrt{3}x\right)$$
$$+ e^{x}\left(\left[C_3 - \sqrt{3}C_2\right]\operatorname{sen}\sqrt{3}x + \left[C_2 + \sqrt{3}C_3\right]\underline{\cos\sqrt{3}x}\right)$$

$$= \left(4C_1 - \tfrac{4}{3} - \tfrac{4}{3}\right)e^{-2x} + \tfrac{8}{3}xe^{-2x}$$
$$+ e^{x}\Big(\left[\sqrt{3}C_3 - 3C_2 + C_2 + \sqrt{3}C_3\right]\cos\sqrt{3}x$$
$$+ \left[-\sqrt{3}C_2 - 3C_3 + C_3 - \sqrt{3}C_2\right]\operatorname{sen}\sqrt{3}x\Big)$$

$$y''(0) = -4$$

$$\therefore \quad -4 = \left(4C_1 - \tfrac{8}{3}\right) + 2\sqrt{3}C_3 - 2C_2 \qquad (3)$$

Agrupando los ecs (1) - (3)

$$\begin{cases} C_1 + C_2 = -5 + \frac{5}{8} = \frac{-40+5}{8} = -\frac{35}{8} & (1) \\ -2C_1 + C_2 + \sqrt{3}C_3 = 3 - \frac{2}{3} - \frac{1}{4} = \frac{9-2}{3} - \frac{1}{4} = \frac{7}{3} - \frac{1}{4} = \frac{28-3}{12} = \frac{25}{12} & (2) \\ 4C_1 - 2C_2 + 2\sqrt{3}C_3 = -4 + \frac{8}{3} = \frac{-12+8}{3} = -\frac{4}{3} & (3) \end{cases}$$

$\longrightarrow$

T3.3 #36 cont.

$$2\{1\}: \quad 2C_1 + 2C_2 = -\frac{35}{4}$$
$$\{2\}: \quad -2C_1 + C_2 + \sqrt{3}C_3 = \frac{25}{12}$$

$$2\{1\}+\{2\}: \quad 0 + 3C_2 + \sqrt{3}C_3 = \frac{-105+25}{12} = \frac{-80}{12} = \frac{-20}{3} \qquad (4)$$

$$\tfrac{1}{2}\{3\}: \quad 2C_1 - C_2 + \sqrt{3}C_3 = -\frac{2}{3}$$

$$\tfrac{1}{2}\{3\}+\{2\}: \quad 0 + 0 + 2\sqrt{3}C_3 = \frac{-2}{3} + \frac{25}{12} = \frac{-8+25}{12} = \frac{17}{12}$$

$$C_3 = \frac{17}{12}\cdot\frac{1}{2\sqrt{3}} = \boxed{\frac{17}{24\sqrt{3}} = C_3}$$

sust. C_3 en (4)

$$3C_2 + \sqrt{3}\left(\frac{17}{24\sqrt{3}}\right) = \frac{-20}{3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & -\frac{35}{8} \\ -2 & 1 & \sqrt{3} & \frac{25}{12} \\ 4 & -2 & 2\sqrt{3} & -\frac{4}{3} \end{array}\right] \text{(1)}$$

$$3C_2 = \frac{-20}{3} - \frac{17}{24} = \frac{-160-17}{24} = \frac{-177}{24}$$

$$\therefore \boxed{C_2 = -\frac{177}{72}}$$

C_2

$$\begin{array}{r} 315 \\ -177 \\ \hline 138 \end{array}$$

sust. C_2 en (1)

$$C_1 - \frac{177}{72} = -\frac{35}{8}$$

$$\therefore C_1 = \frac{177-315}{72} = \frac{-138}{72} = -\frac{69}{36} \rightarrow \boxed{C_1 = -\frac{23}{12}}$$

$\therefore$ la sol. del PVI es:

$$\boxed{y = -\frac{23}{12}e^{-2x} + e^{x}\left(-\frac{177}{72}\cos\sqrt{3}x + \frac{17}{24\sqrt{3}}\operatorname{sen}\sqrt{3}x\right) + \frac{1}{4}x - \frac{5}{8} + \frac{2}{3}xe^{-2x}}$$

$$C_1 = -\frac{23}{12} \qquad C_2 = -\frac{177}{72} \qquad C_3 = \frac{17}{24\sqrt{3}}$$

ver pr36p158.ggb

(1) $C_1 = -1.916\bar{6}$; $C_2 = -2.458\bar{3}$; $C_3 = 0.4089$ (en Scilab)