M5. 2010.07.26 (A)

Ejemplo 1 (pp. 319-320) <u>Complemento</u> ▽:

claro: $X_1 = X_1(s)$ y $X_2 = X_2(s)$:

$$\boxed{\begin{array}{lr} (s^2+10)X_1 - 4X_2 = 1 & (1) \\ -4X_1 + (s^2+4)X_2 = -1 & (2) \end{array}}$$

Si $\{1\}' \leftarrow \dfrac{4}{s^2+10}\{1\}$

$$4X_1 - \frac{16X_2}{s^2+10} = \frac{4}{s^2+10} \qquad (3) = \{1\}'$$

$\{4\} \leftarrow \{1\}' + \{2\}$

$$\left(s^2+4 - \frac{16}{s^2+10}\right)X_2 = \frac{4}{s^2+10} - 1 \qquad (4)$$

$\{5\} \leftarrow (s^2+10)\{4\}$

$$[(s^2+4)(s^2+10) - 16]\,X_2 = 4 - (s^2+10) \qquad (5)$$

$$\therefore \quad X_2 = \frac{-(s^2+6)}{(s^2+4)(s^2+10)-16} = \frac{-6-s^2}{s^4+14s^2+40-16}$$

$$X_2 = \frac{-6-s^2}{s^4+14s^2+24} = \frac{-6-s^2}{(s^2+12)(s^2+2)} \qquad (6)$$

Por fracc. parciales:

$$X_2 = \frac{As+B}{s^2+12} + \frac{Cs+D}{s^2+2} \qquad (7)$$

Es decir:

$$-6-s^2 = (As+B)(s^2+2) + (Cs+D)(s^2+12)$$

$\longrightarrow$

▽ Ver planteamiento en texto Zill.

(complemento)

$\boxed{\text{cont.}}$ ejemplo 1. (pp. 319-320)

$$-1 = B + D \quad (8)$$

$$0 = 2A + 12C \quad (9)$$

$$-6 = 2B + 12D \quad (10)$$

$$0 = A + C \quad (11)$$

Combinando (11) * (-2) + (9)

$$\begin{array}{rl} 0 = & 2A + 12C \\ 0 = & -2A - 2C \\ \hline 0 = & 10C \end{array} \rightarrow \boxed{C = 0}$$

sust. $C=0$ en (11) obtenemos $\boxed{A=0}$

Combinando ahora (8) * (-2) + (10)

$$\begin{array}{rl} 2 = & -2B - 2D \\ -6 = & 2B + 12D \\ \hline -4 = & 10D \end{array} \rightarrow \boxed{D = -\frac{2}{5}}$$

sust. $D = -\frac{2}{5}$ en (8)

$$B = -1 - D = -1 + \frac{2}{5} = -3/5$$

$$\boxed{B = -\frac{3}{5}}$$

$$\therefore \quad X_2 = \frac{-3/5}{s^2+12} + \frac{-2/5}{s^2+2} \quad (12)$$

$$= -\frac{3}{5} \cdot \frac{1}{\sqrt{12}} \cdot \frac{\sqrt{12}}{s^2+12} - \frac{2}{5} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{s^2+2}$$

$$X_2 = -\frac{3}{5}\, \frac{1}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \left(\frac{\sqrt{12}}{s^2+12} \right) - \frac{2}{5} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \left(\frac{\sqrt{2}}{s^2+2} \right)$$

← aplicando Laplace.

$$\boxed{x_2(t) = -\frac{\sqrt{3}}{10} \operatorname{sen} \sqrt{12}\, t - \frac{\sqrt{2}}{5} \operatorname{sen} \sqrt{2}\, t} \quad (13)$$

(complemento)
$\boxed{\text{cont.}}$ ejemplo 1 (pp. 319-320)

de (2)
$$X_1 = \frac{-1-(s^2+4)X_2}{-4}$$

$$X_1 = \frac{1}{4} + \frac{s^2+4}{4} X_2 \qquad (14)$$

sust. (12) en (14)
$$X_1 = \frac{1}{4} + \frac{s^2+4}{4}\left[\frac{-3/5}{s^2+12} + \frac{-2/5}{s^2+2}\right]$$

$$X_1 = \frac{1}{4} - \frac{3}{20}\cdot\frac{s^2+4}{s^2+12} - \frac{2}{20}\cdot\frac{s^2+4}{s^2+2} \qquad (15)$$

pero como:
$$\frac{s^2+4}{s^2+12} = \frac{s^2+4+8-8}{s^2+12} = \frac{s^2+12-8}{s^2+12} = 1 - \frac{8}{s^2+12}$$

y
$$\frac{s^2+4}{s^2+2} = \frac{s^2+4-2+2}{s^2+2} = \frac{s^2+2+2}{s^2+2} = 1 + \frac{2}{s^2+2}$$

podemos reescribir (15) como:
$$X_1 = \frac{1}{4} - \frac{3}{20}\left(1 - \frac{8}{s^2+12}\right) - \frac{1}{10}\left(1 + \frac{2}{s^2+2}\right)$$

$$X_1 = \underbrace{\left(\frac{1}{4} - \frac{3}{20} - \frac{1}{10}\right)}_{0} + \frac{24}{20}\cdot\frac{1}{\sqrt{12}}\left(\frac{\sqrt{12}}{s^2+12}\right) - \frac{2}{10}\cdot\frac{1}{\sqrt{2}}\ \frac{\sqrt{2}}{s^2+2} \quad ; \sqrt{12} = 2\sqrt{3}$$

aplicando Laplace:
$$x_1(t) = \frac{12}{10}\cdot\frac{1}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}\ \text{sen}\ \sqrt{12}\,t - \frac{2}{10}\cdot\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\ \text{sen}\ \sqrt{2}\,t$$

$$\boxed{x_1(t) = \frac{\sqrt{3}}{5}\ \text{sen}\ 2\sqrt{3}\,t - \frac{\sqrt{2}}{10}\ \text{sen}\ \sqrt{2}\,t}$$

ver ej1p319.ggb